Prof. Vidya Sagar
Department of Physics
Raja Singh College, Siwan

B.Sc (Part-1) Hons
Paper-II

Theory of determination of thermal conductivity of a metal rod by periodic flow method.

Periodic flow of heat:— When one end of a bar is alternately heated and cooled, the heat flows through the bar periodically. Then the temprature at any given point on the bar also varies periodically.

Let us assume that temperature at the hot end of a bar, insulated from surroundings, varies simple harmonically, and is given by

$$\theta = \theta_0 \cos\omega t \quad \text{———} (1)$$

Where θ_0 is the temprature-amplitude. Let the direction of heat-flow be along the x-axis. Then the Fourier's differential equation will be

$$\frac{d\theta}{dt} = K\frac{d^2\theta}{dx^2} \quad \text{———} (2)$$

where K is diffusivity

To solve the equation, let us try a solution

$$\theta = Ae^{\alpha x + i\beta t} \quad \text{———} (3)$$

Where A, α, β are constants and $i = \sqrt{-1}$

This gives

$$\frac{d\theta}{dt} = Ae^{\alpha x + i\beta t}(i\beta) \quad \text{and}$$

$$\frac{d^2\theta}{dx^2} = Ae^{\alpha x + i\beta t}(\alpha^2)$$

Substituting in eqn (2) we get

$$i\beta = K\alpha^2$$

$$\therefore \alpha = \pm\sqrt{\frac{j\beta}{K}}$$

Now $(1+j)^2 = 2j$ and so $\sqrt{j} = \frac{1}{\sqrt{2}}(1+j)$. Hence

$$\alpha = \pm\sqrt{\left(\frac{\beta}{2K}\right)}(1+j).$$

Substituting this value in eqn(3), we get

$$\theta = Ae^{\pm\sqrt{\frac{\beta}{2K}}(1+j)x + j\beta t}$$

When $x = \infty$ (at cold end), $\theta \neq \infty$. Hence only the negative sign is admissible.

Separating the real and imaginary part we get

$$\theta = Ae^{-\sqrt{\frac{\beta}{2K}}x + j\left(\beta t - \sqrt{\frac{\beta}{2K}}x\right)}$$

$$= Ae^{-\sqrt{\frac{\beta}{2K}}x}\left\{\cos\left(\beta t - \sqrt{\frac{\beta}{2K}}x\right)\right.$$

Rejecting the imaginary term, we have

$$\theta = Ae^{-\sqrt{\frac{\beta}{2K}}x}\cos\left(\beta t - \sqrt{\frac{\beta}{2K}}x\right)$$

At $x = 0$ (hot end), this gives $\theta = A\cos\beta t$. Comparing this with equation (1) we have

$A = \theta_0$ and $\beta = \omega$. Hence we have

$$\theta = \theta_0 e^{-\sqrt{\frac{\omega}{2K}}x}\cos\left(\omega t - \sqrt{\frac{\omega}{2K}}x\right)$$

This represents heat wave travelling with velocity

$$\sqrt{2\omega K} = \sqrt{\frac{4\pi K}{T}}$$

It shows that a particular point along the bar (x = Constant) the temprature θ varies harmonically with time, the period being that of the heat Source. It further shows that the amplitude of temprature oscillations diminishes exponentially as the distance x along the bar increases and becomes negligible at a sufficient distance.